Design of Coupled Resonators Group Delay Equalizers

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***Abstract* — A systematic design procedure for coupled resonators cavity group delay equalizers is presented. The procedure consists of solving the approximation problem by optimization and solving the synthesis problem. The error function for the optimization is computed from filter's group delay and the zeros and poles of the input impedance of the equalizer. Convergence of the optimization is fast and insensitive to the initial guess even when the number of resonators is large. Two examples together with experimental results are presented to show the powerfulness and effectiveness of the proposed procedure.**

I. INTRODUCTION

Microwave bandpass networks together with equalization circuitry are essential components in modern communication systems such as satellite communication systems. Conventional realizations of group delay equalizers are mostly limited to all-pass C-sections (all-pass first-order) and all-pass D-section (all-pass second-order) networks cascaded with circulators or 3-dB hybrids. Graphical methods are used to determine the suitable locations of zeros of an equalizer followed by Richards' synthesis procedure to complete the design [1], [2]. This approach works well when the amount of equalization required is small. When a larger amount of equalization is needed, several C- or D-section equalizers may be required in cascade, which makes the design of such equalizers difficult.

A design method for equalizers with multiple coupled cavities was first presented in 1982 [3]. Direct network optimization has been used in [3], where the coupling matrix elements are the optimization variables and the difference between the group delay response of the equalizer and the frequency specification mask is the basis for the objective error function. Optimization routines or available commercial software packages may be used to solve for all the optimization variables by minimizing the objective error function over the frequency band of interest. The design method proposed in [3] showed two major advantages over the conventional approach: (1) a single equalizer with multiple poles is able to perform a larger degree of equalization that provides a considerable hardware weight reduction over the conventional approach of using several cascaded C- or D-sections and (2) equalizer parameters are directly generated through the optimization process which eliminates the synthesis step, making the design procedure straightforward.

However, the approach proposed in [3] is usually inefficient and often results in non-optimum (local minimum) solutions. Moreover, the convergence of numerically minimizing the objective error function will depend strongly on the initial guess of the equalizer parameters (optimization variables), especially when the number of cavities is large. In this paper, a systematic design procedure is presented. This new and powerful design procedure basically consists of two separate steps : (1) solving the approximation problem through numerical optimization and (2) solving the synthesis problem. The objective error function used for solving the approximation problem is based on evaluating the group delay response using the zeros and poles of the input impedance function of the structure. Convergence of the optimization of proposed design method is fast and nearly independent of the initial guess of zeros and poles locations even when the number of the cavities is large. Once the locations of zeros and poles are determined, a synthesis procedure is then carried out to complete the design. Detailed design procedure will be presented in Section II including the problem statement and circuit analysis. A straightforward synthesis procedure is presented in Section III. To show the powerfulness of the proposed design method, two numerical examples are given in section IV, with experimental results of an 8-pole elliptic function filter and 3-pole equalizer included.

II. THE APPROXIMATION PROBLEM

Consider a circulator coupled filter-equalizer network as shown in Fig. 1. Since the total group delay of the network is the sum of the group delay of the filter and the reflection delay of the equalizer, ideally the delay characteristics of the equalizer should be the inverse of the

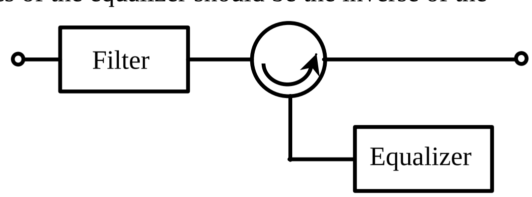

Fig. 1. Circulator coupled filter-equalizer network.

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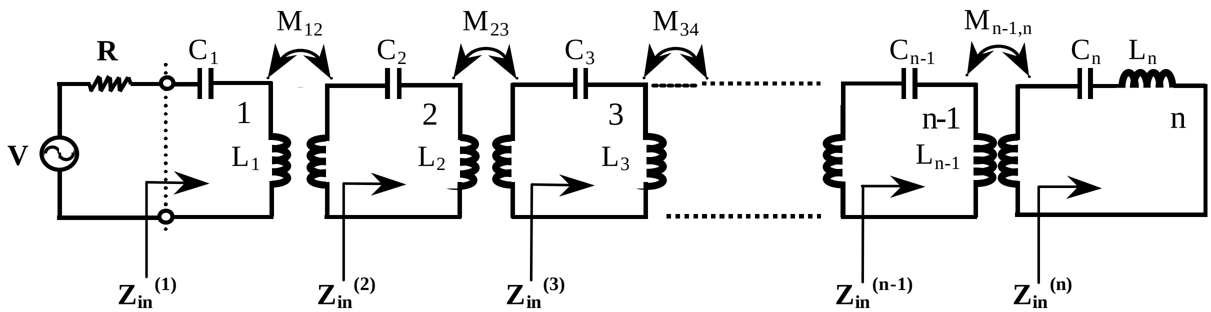

Fig. 2. Lumped circuit representation for an n-cavity narrow-band equalizer.

filter's to compensate for the steep change at the edges of the band. The delay of the equalizer is the delay characteristic of the reflection coefficient of a short-circuited coupled resonators filter [4]-[6].

Fig. 2 shows the equivalent lumped circuit representation for a short-circuited coupled resonators narrow band equalizer. Although this equivalent circuit is accurate only over a narrow bandwidth (<20%), it is usually sufficient in most applications where large amount of equalization is only required over a narrow bandwidth. The input impedance of the equalizer can be obtained as:

$$Z_{in}^{(1)}(w) = j\frac{Z_{o1}}{ww_{o1}}\frac{P_1(w^2)}{Q_1(w^2)} \qquad (1)$$

$P_1(w^2)$ and $Q_1(w^2)$ are monic polynomials of order n and (n-1), respectively; $w_{o1} = 1/\sqrt{L_1C_1}$ is the resonant frequency of the first resonator and $Z_{o1} = \sqrt{L_1/C_1}$ is the characteristic impedance of the first resonator. The input reflection coefficient of the equalizer can be readily expressed as:

$$r_{in}(w) = \frac{Z_{in}^{(1)}(w) - R}{Z_{in}^{(1)}(w) + R} = \left|r_{in}(w)\right|e^{jf(w)} \qquad (2)$$

$$f(w) = -2\tan^{-1}\left[\frac{Z_{o1}P_1(w^2)}{ww_{o1}RQ_1(w^2)}\right] \qquad (3)$$

The group delay of the equalizer, by definition, is the negative derivative of $f(w)$ with respect to w. Noting the fact that the input impedance (1) is purely reactive, the group delay of the equalizer can then be easily derived as:

$$t_E(w) = -\frac{df(w)}{dw} = -\frac{1}{j}\left[\frac{d}{dw}\ln(r_{in}(w))\right] = \frac{2w_{o1}\frac{R}{Z_{o1}}\left[w\left(\frac{P_1'}{P_1} - \frac{Q_1'}{Q_1}\right) - 1\right]}{\left(\frac{P_1}{Q_1}\right) + \left(\frac{ww_{o1}R}{Z_{o1}}\right)^2\left(\frac{Q_1}{P_1}\right)} \qquad (4)$$

, where P_1' and Q_1' denote the derivatives of P_1 and Q_1 with respect to w, respectively. When the network shown in Fig. 2 is near the optimum approximation to the requirement, $P_1(w^2)$ and $Q_1(w^2)$ can be expressed as: [4],[6]

$$P_1(w^2) = \prod_{i=1}^{n}(w^2 - w_{zi}^2) \qquad (5a)$$

$$Q_1(w^2) = \prod_{j=1}^{n-1}(w^2 - w_{pj}^2) \qquad (5b)$$

In the above expressions, $w_{zi}\,(i = 1,2,\ldots,n)$ and $w_{pj}\,(j = 1,2,\ldots,n-1)$ are zeros of P_1 and Q_1, corresponding to the zeros and poles of the input impedance function respectively.

The approximation problem can be stated as follows: Given a certain requirement on the group delay (frequency specification mask) to be met by the proposed network, determine the locations of the zeros and poles of the input impedance function of the equalizer that realize the desired group delay characteristic. The approximation problem is solved numerically through optimization. The optimization procedure starts by an initial guess for the locations of zeros and poles. A simple initial guess will be taking the initial placement of the zeros and poles to be alternating, and equally distributed over the desired bandwidth. The objective error function is defined as:

$$Errf(\bar{x}) = \sum_i \left|t_F(w_i) + t_E(w_i) - t_{\min} - T_{mask}(w_i)\right|^2,$$

$$\bar{x} = (w_{z1}, w_{z2}, \ldots, w_{zn}, w_{p1}, w_{p2}, \ldots, w_{p,n-1}, R/Z_{o1}) \qquad (6)$$

$$i \in \left\{t_F(w_i) + t_E(w_i) - t_{\min} - T_{mask}(w_i) > 0\right\}$$

with t_F, t_E : group delay of filter and equalizer,
$t_{\min}$: the minimum of total group delay (constant),
T_{mask} : required frequency specification mask.

In the above expression, t_E and $t_{\min}$ are evaluated from the current placement of the zeros and poles. A standard gradient constrained search minimization algorithm is used to minimize the objective error function since the locations of the zeros and poles are known to be interlaced in the vicinity of the band of the network, i.e., $w_{z1} < w_{p1} < w_{z2} < w_{p2} < \cdots < w_{z,n-1} < w_{p,n-1} < w_{z,n}$. Convergence of the minimization is fast resulting from the constrained nature among the optimization variables. In most cases, we have symmetry with respect to the center frequency which speeds up convergence dramatically since only half the number of optimization variables is necessary for optimization.

III. Synthesis Of Equalizer

After solving the approximation problem, the exact locations of the zeros and poles that realize the required group delay characteristic of the network are known. To complete the design, a synthesis procedure is adopted to extract the equalizer parameters from the knowledge of the locations of zeros and poles of the network.

Using the same notations as defined in Fig. 2, the input impedance at loop i can be obtained as:

$$Z_{in}^{(i)}(w) = j\frac{Z_{oi}}{ww_{oi}}\frac{P_i(w^2)}{Q_i(w^2)} \qquad ,i = 1,2,\dots n \tag{7}$$

where $w_{oi} = 1/\sqrt{L_i C_i}$ is the resonant frequency of resonator i and $Z_{oi} = \sqrt{L_i / C_i}$ is the characteristic impedance of the i^{th} resonator. The monic polynomials $P_i(w^2)$ and $Q_i(w^2)$ are expressed as: [6]

$$P_i(w^2) = \sum_{t=0}^{n-i+1} c_t^{(i)}(w^2)^t = \prod_{t=1}^{n-i+1}(w^2 - w_{zt}^{(i)2}), i = 1,2,\dots n \tag{8}$$

$$Q_i(w^2) = \sum_{q=0}^{n-i} d_q^{(i)}(w^2)^q = \prod_{q=1}^{n-i}(w^2 - w_{pq}^{(i)2}), i = 1,2,\dots n \tag{9}$$

Since the circuit model of Fig. 2 is an accurate representation over a narrow bandwidth as discussed previously, the coupling coefficients between two adjacent resonators $k_{i,i+1}$ can be modeled as frequency independent reactance and can be defined as:

$$k_{i,i+1}^2 = \frac{M_{i,i+1}^2}{Z_{oi}Z_{oi+1}} \qquad ,i = 1,2,\dots n-1 \tag{10}$$

The coupling bandwidth $m_{i,i+1}$ is then defined as:

$$m_{i,i+1}^2 = w_{oi}w_{oi+1}\frac{M_{i,i+1}^2}{Z_{oi}Z_{oi+1}} \qquad ,i = 1,2,\dots n-1 \tag{11}$$

and is the coupling coefficient in frequency unit. The following recursive relations can be derived from basic circuit theory:

$$P_{i+1}(w^2) = Q_i(w^2) \qquad ,i = 1,2,\dots n-1 \tag{12}$$

$$(w^2 - w_{oi}^2)P_{i+1}(w^2) - P_i(w^2) = m_{i,i+1}^2 w^2 Q_{i+1}(w^2) \qquad ,i = 1,2,\dots n-1 \tag{13}$$

$$w_{oi}^2 = \frac{\prod_{t=1}^{n-i+1} w_{zt}^{(i)2}}{\prod_{q=1}^{n-i} w_{pq}^{(i)2}} \qquad ,i = 1,2,\dots n \tag{14}$$

$$m_{i,i+1}^2 = \sum_{t=1}^{n-i+1} w_{zt}^{(i)2} - \sum_{q=1}^{n-i} w_{pq}^{(i)2} - w_{oi}^2 \qquad ,i = 1,2,\dots n-1 \tag{15}$$

(14) and (15) give explicit relations between the equalizer parameters and the locations of zeros and poles. Thus, synthesis of the equalizer can be completed from (12)~(15) after solving for the locations of zeros and poles from the approximation problem.

IV. Numerical Examples

A computer program has been developed to solve the approximation problem through optimization and perform the synthesis of the equalizer as described. Convergence of the optimization is fast and in all cases tested is nearly independent of the initial placement of the zeros and poles. In contrast, direct network optimization using an error function based on the difference between the mask and response was slow, often did not converge to any acceptable solution, and in all cases required an initial guess whose response was close to the desired one in order to converge, especially when the number of cavities is large.

Many design examples have been run to verify and test the program. Two of these examples are summarized in Table I. In both cases, the initial placement of the zeros and poles is taken to be equally distributed in the vicinity of the bandwidth of the network. Fig. 3(a) and (b) show the simulated response of the total group delay (filter and equalizer) after solving the approximation problem. The frequency specification masks are also included in both figures. Fig. 4 shows the in-band group delay of the overall structure including the filter and the equalizer for design (a). Two different circulators are used for cascading the filter and the equalizer. The thinner line denotes the one using an ideal circulator with infinite isolation while the thicker one denotes the one using a non-ideal circulator with 28dB isolation. It is clear that the isolation of the circulator will have pronounced effect on the ripple of the final response.

A 3-cavity equalizer has been built according to design (a) to equalize an 8-pole elliptic function filter. Fig. 5 shows the measurement results. A circulator with 28dB isolation was used in the experiment, and the measured results agreed well with simulation.

TABLE I

Summary Of Two Design Examples

Network Structure	Start Error	Final Error	Optimized Response
(a) 8-pole filter cascaded with n=3 equalizer	3.1e04	1.3e-5	Fig. 3(a)
(b) 16-pole filter cascaded with n=12 equalizer	1.4e06	3.5e-4	Fig. 3(b)

V. Conclusion

A powerful and systematic method for the design of coupled cavity group delay equalizers is introduced. This method consists of solving the approximation problem by optimization and synthesis of the network parameters. The presented method is insensitive to the initial guess of the locations of zeros and poles, and converges fast even when the number of cavities is large. Typical examples together with measurement results of practical equalizers are given which show the effectiveness and powerfulness of the method.

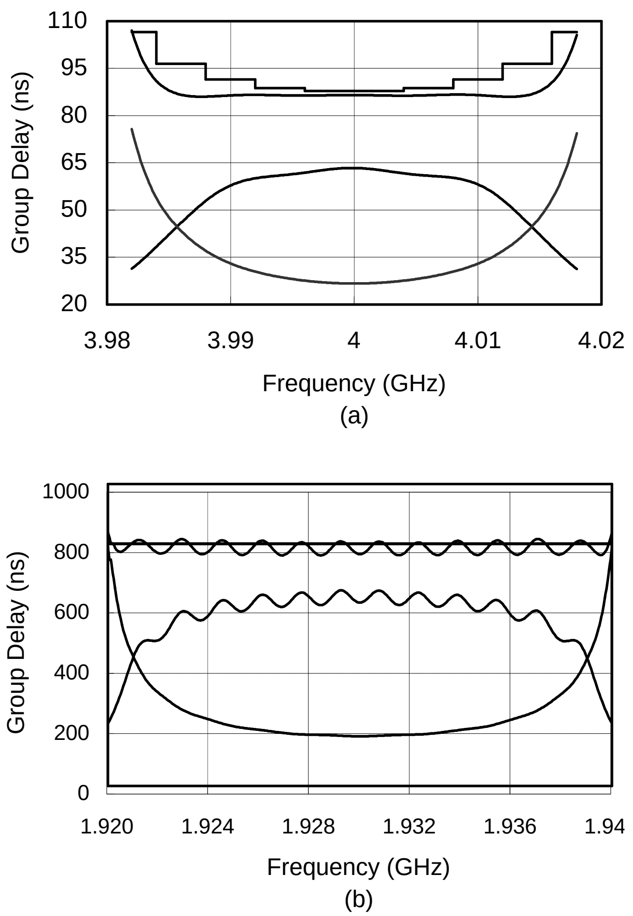

Fig. 3. Group delay response after optimization for design (a) and (b), respectively. In both plots, the four curves denote, from top to bottom, the spec. mask, total group delay, group delay of equalizer and group delay of filter.

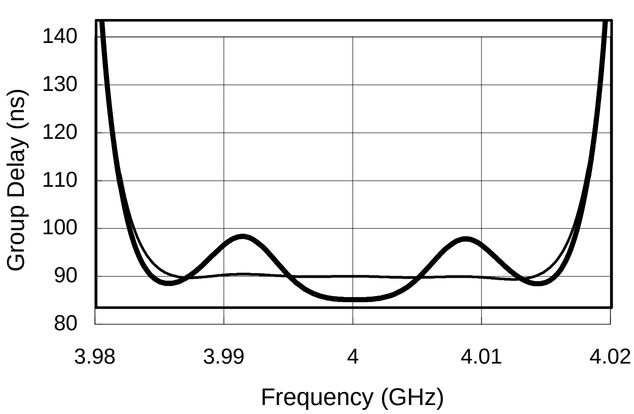

Fig. 4. The simulated final response of in-band group delay for design (a). The unloaded Q for the equalizer is set to be 3500. The responses for using both ideal (thinner line) and non-ideal (thicker line) circulator are included for comparison.

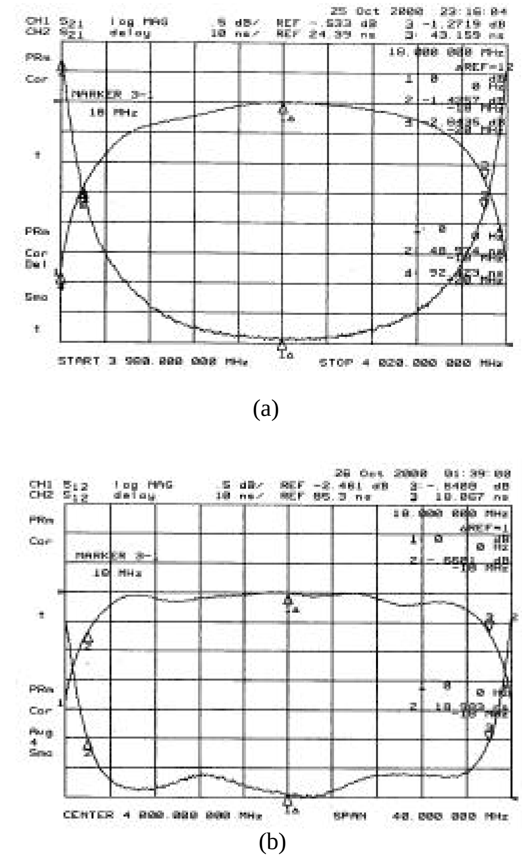

Fig. 5. Measured group delay and insertion loss of (a) filter only and (b) filter and equalizer for an 8-pole elliptic function filter and a 3-cavity equalizer (design (a)). The non-idealness of the circulator used contributes for the ripple of the response.

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